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IMPULSE-FUNCTION METHOD FOR HEAT-TRANSFER DYNAMICS IN A CHANNEL

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A method of solving the boundary problem for heat-transfer dynamics in a channel is proposed; the problem is reduced to integral equations of Volterra type.

Nonsteady one-dimensional motion of heat carrier in a heated channel is considered. The problem is to determine the change in the parameters (temperature, flow rate, pressure) due to perturbation of the external conditions. The change in the flow rate and pressure at the channel inlet are related by the boundary conditions for the equation of motion. In most of the known works, the conditions at the right-hand boundary were either completely disregarded [1], or else were assumed to affect only the pressure deviation at the inlet and the flow rate was assumed to be given [2]. If conditions are specified at both boundaries, it is necessary to solve a boundary problem for the system of equations describing the heat transfer and hydrodynamics. A more complete formulation of the problem is possible if numerical methods are used for the direct integration of the differential equations, but to date this approach has been used mainly in the context of scientific research because the computational algorithms are too complex for use in engineering practice.

The present work describes a method by which, in the linear case, the boundary problem can be reduced to two integral Volterra equations of the second kind of convolution type; analytic expressions are obtained for the impulse function relating the changes in input and output parameters. Computer solution of the integral equations is straightforward.

Taking the equations of statics into account [3], a linearized system of conservation equations may be written for the parameter deviations:

$$\frac{\partial \Delta D}{\partial z} + f\frac{\partial \Delta \rho}{\partial \tau} = 0, \tag{1}$$

$$D_0\frac{\partial \Delta i}{\partial z} + f\rho_0\frac{\partial \Delta i}{\partial \tau} + \frac{\partial i_0}{\partial z}\Delta D = \Delta\alpha h(\theta_0 - t_0) + \alpha_0 h(\Delta\theta - \Delta t), \tag{2}$$

$$\Delta q - g_w c_w\frac{\partial \Delta\theta}{\partial \tau} = \Delta\alpha h(\theta_0 - t_0) + \alpha_0 h(\Delta\theta - \Delta t), \tag{3}$$

$$\Delta p_1 - \Delta p = \frac{2\delta p_0}{D_0}\Delta D - \frac{\delta p_0}{\rho_0}\Delta\rho,$$

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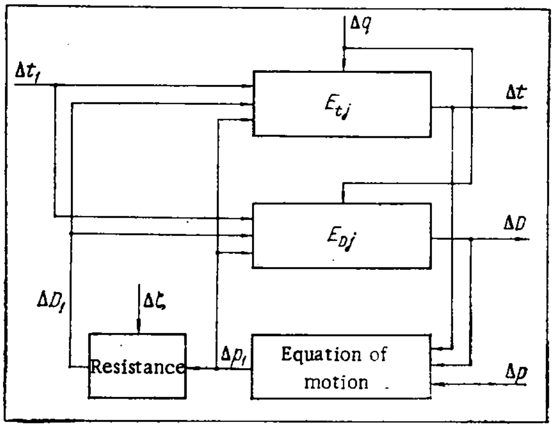


Fig. 1. Structural diagram of calculation.

$$\Delta\rho = \frac{\partial\rho}{\partial t}\Delta t + \frac{\partial\rho}{\partial p}\Delta p, \quad \Delta i = \frac{\partial i}{\partial t}\Delta t + \frac{\partial i}{\partial p}\Delta p. \tag{4}$$

Here the heat-transfer coefficient is assumed to be a function only of the flow rate, and the equation of motion is considered for resistance concentrated at a point. The heat flow is independent of the space coordinate $\Delta q = \Delta q(\tau)$, and the initial value of the deviations is zero.

The choice of boundary conditions is based on the assumption that the medium leaves the source at constant pressure and passes through a controlled resistance to the input of a channel; the pressure at the channel output is independent of the processes that occur in the channel.* Thus, the changes in flow rate and pressure at the input are related by the controlled-resistance equation

$$\Delta p_1(\tau) + \frac{D_0^2}{\rho_1}\Delta\zeta(\tau) + \frac{2D_0\zeta}{\rho_1}\Delta D_1(\tau) = 0. \tag{5}$$

The deviations Δt_1, $\Delta\zeta$, Δq, and Δp may vary arbitrarily with time.

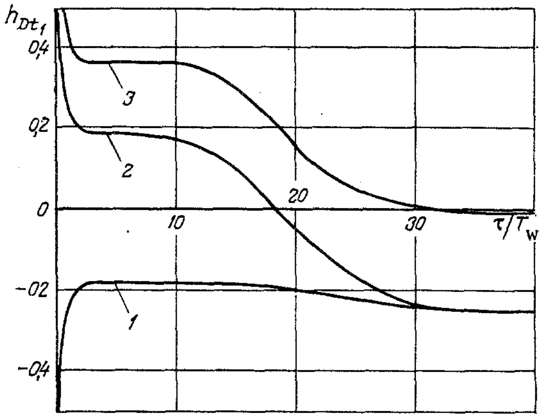
Certain of the relations in Eqs. (1)-(5) are weak. Experimental data and calculations show that in the case of slightly compressible flow the effect of density variations on the temperature may be neglected in Eq. (2) [so that $\Delta D(z, \tau) = \Delta D_1(\tau)$]; the temperature deviation may then be found without considering the continuity equation, but taking into account the flow-rate variation in the input cross section. Slight flow-rate oscillations arising as a result of flow compressibility may be determined subsequently from Eq. (1) using the results obtained for the deviation $\Delta t(z, \tau)$. Because the thermophysical flow properties depend very little on the pressure variation in the transient process, the equation of motion need only be considered when the pressure deviations at the channel boundaries are to be determined. Since the equation of motion forms an internal feedback in the channel, it may be broken, in complete accordance with the principles of control theory, by detaching Eq. (4) from the system. Subsequently, by finding the dynamic characteristics of this broken circuit, this feedback connection may be restored.

In accordance with the discussion above, the system in equations (1)-(5) is divided into three parts, combined in the structural diagram of the calculation (Fig. 1). In the diagram, the propagation of perturbations from input to output is expressed by an impulse transient function using a Duhamel integral (convolution). The boundary conditions on Eq. (1)-(3) required to determine the impulse functions relating the temperature perturbations at the input to the temperature and flow-rate variations at an arbitrary cross section [$E_{tt_1}(t)$ and $E_{Dt_1}(\tau)$] are written in the form

$$\Delta t_1(\tau) = \delta(\tau), \quad \Delta D_1(\tau) = \Delta p_1(\tau) = \Delta q(\tau) = 0,$$

where $\delta(\tau)$ is a Dirac delta function. The impulse functions for the other perturbations (ΔD_1, Δp_1, Δ_q) are found analogously.

* In this example, the simplest possible specification of the boundary conditions is adopted; this is not a limitation of the method.



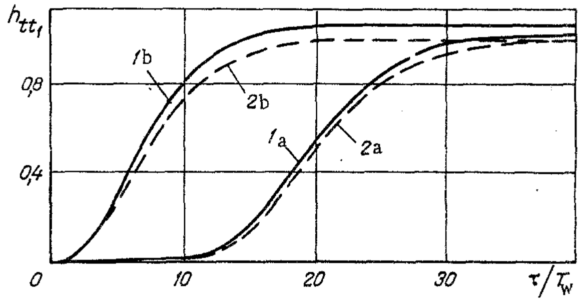


Fig. 2 Fig. 3

Fig. 2. Normalized startup curves for flow rate $h_{Dt_1} = \Delta D/\varkappa\Delta t_1$, $\varkappa = -(D_0/\rho_0)(\partial\rho/\partial t)$: 1) at input; 2) at output; 3) at output without taking equation of motion into account; D, kg/sec; t, °C; ρ, kg/m³; τ, sec; T_W, sec.

Fig. 3. Normalized startup curves for temperature $h_{tt_1} = \Delta t/\Delta t_1$ with (1) and without (2) taking equation of motion into account; a) water; b) steam.

The solution employs integral Laplace transforms. The correspondence principle is used in obtaining the impulse transient function [3, 4]. For example, in the case of a temperature perturbation

$$E_{tt_1} = \frac{\xi}{T_w \eta} V_{2,0} + \delta(\tau - \tau_{\tau\rho}) \exp(-\xi),$$

$$E_{Dt_1} = \varkappa \left[\delta(\tau) - E_{tt_1} - \frac{1}{T_B} (\exp(-s_0\tau) - V_{1,c}) \right],$$

where $V_{2,0}$ and $V_{1,c}$ are special functions of Bessel type [4]. By means of the impulse functions it is possible to find the temperature and flow-rate distributions for input perturbations $\Delta j(\tau)$ of arbitrary form

$$\Delta t(\tau) = \sum_{j=1}^{4} \int_0^{\tau} E_{tj}(\tau - x) \Delta j(x)\, dx, \tag{6}$$

$$\Delta D(\tau) = \sum_{j=1}^{4} \int_0^{\tau} E_{Dj}(\tau - x) \Delta j(x)\, dx, \quad j - t_1,\ D_1,\ p_1,\ q. \tag{7}$$

It is necessary to add to Eqs. (6) and (7) the equation of motion in the channel, taking into account the equation of state

$$\left(1 + \frac{\delta p_0}{\rho_0} \frac{\partial\rho}{\partial p}\right) \Delta p_1(\tau) = \Delta p(\tau) - \frac{\delta p_0}{\rho_0} \frac{\partial\rho}{\partial t} \Delta t(\tau) + \frac{2\delta p_0}{D_0} \Delta D(\tau), \tag{8}$$

and Eq. (5). Eliminating $\Delta p_1(\tau)$ and $\Delta D_1(\tau)$ from these equations leads to two Volterra integral equations of the second kind for $\Delta t(\tau)$ and $\Delta D(\tau)$:

$$\Delta t(\tau) = F_t(\tau) + \int_0^{\tau} [B_t \Delta t(x) + B_D \Delta D(x)] N_t(\tau - x)\, dx,$$

$$\Delta D(\tau) = F_D(\tau) + \int_0^{\tau} [B_t \Delta t(x) + B_D \Delta D(x)] N_D(\tau - x)\, dx.$$

Here

$$N_t(\tau - x) = E_{tD_1}(\tau - x) B_p + E_{tp_1}(\tau - x);$$

$$N_D(\tau - x) = E_{DD_1}(\tau - x) B_p + E_{Dp_1}(\tau - x)$$

are the integral kernels; also

$$F_t(\tau)=\int_0^\tau [E_{tt_1}(\tau-x)\Delta t_1(x)+E_{tq}(\tau-x)\Delta q(x)+N_t(\tau-x)\Delta p(x)]\,dx;$$

$$F_D(\tau)=\int_0^\tau [E_{Dt_1}(\tau-x)\Delta t_1(x)+E_{Dq}(\tau-x)\Delta q(x)+N_D(\tau-x)\Delta p(x)]\,dx$$

are inhomogeneous terms; and

$$B_t=-\frac{1}{C}\frac{\delta p_0}{\rho_0}\frac{\partial\rho}{\partial t};\quad B_D=\frac{2\delta p_0}{D_0 C};\quad B_p=-\frac{\rho_1}{2\zeta D_0};$$

$$C=1+\frac{\delta p_0}{\rho}\frac{\partial\rho}{\partial p}$$

are coefficients, For simplicity, let $\Delta\zeta=0$.

Relations of input—output type are very convenient for the construction of structural diagrams for the calculation of various technological processes. By means of a structural diagram, a complex process may be represented as a combination of elementary processes, readily susceptible to theoretical analysis. This significantly increases the efficiency of computer use, because of the convenient organization of the information flow.

Curves of flow-rate deviation for a step change in water temperature at the input are shown in Fig. 2; also, for comparison, theoretical curves calculated without taking into account the conditions on the right-hand boundary (Cauchy problem) are shown. The area between curves 1 and 2 characterizes the change in mass filling of the channel due to the arrival of hotter water. Since $\varkappa$ is small, the absolute value of the deviation is also small, but the change in the final level of the flow rate affects the temperature deviation both in the channel under consideration (Fig. 3) and in those following it. In a complex heat-exchanger system, such as a power steam generator, small changes in flow rate may have a cumulative effect that is significant in practical calculations.

If the external perturbation takes up a steady value with time, integration of Eqs. (6) and (7) between infinite limits leads to algebraic equations in which the deviations Δt and ΔD are related to the perturbations by means of an amplification factor. Joint solution of these equations and Eqs. (5) and (8) leads to an expression for finite steady temperature and flow-rate deviations. Thus, if the input temperature of the flow changes by Δt_1, the temperature deviation at the output cross section may be determined from the expression

$$\Delta t=\Delta t_1+\frac{A\Delta t_1}{2\delta p_0/D_0-C/B_p-A},$$

where

$$A=\frac{t_{10}-t_0}{D_0}\frac{\delta p_0}{\rho_0}\frac{\partial\rho}{\partial t}.$$

The first term in this expression is completely determined by the magnitude of the applied perturbation; the second characterizes the extent to which the transmission of the perturbation through the channel is undetected. Steady deviations of the parameters may be determined analogously for all possible perturbations.

NOTATION

D, flow rate; i, enthalpy; t, flow temperature; ρ, density; f, cross section; α, heat-transfer coefficient; θ, wall temperature; q, g, h, heat flow, mass, and internal surface per unit length; c, specific heat; p, pressure; δp, pressure difference; ζ, reduced controlled-resistance coefficient; τ, time; z, coordinate; Δ, deviation. Indices: 0, initial; 1, output; w, wall.

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NEW METHOD FOR NONSTEADY-HEAT-TRANSFER INVESTIGATIONS IN A THERMAL AERODYNAMIC TUBE

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A new method of thermal testing and processing the resulting data is developed on the basis of Tikhonov's regularization method. Nonsteady heat transfer is investigated on an elongated model.

A new method of treating the results of thermal tests in an aerodynamic tube has been developed recently [2, 3] based on A. N. Tikhonov's regularization technique for the solution of inverse heat-conduction problems. This method can be used for heat-transfer investigations in significantly nonsteady conditions and hence, in contrast to existing thermal-testing methods using regular heating conditions, it is unnecessary to wait some time after tube startup so as to ensure that the model is introduced into steady flow. The model may be introduced into the working part of the tube earlier, and flow with a uniform field over practically the whole of the characteristic rhomb of the nozzle may be used. As a result, a comparatively small working part of the tube may provide a large value of Re, calculated over the length of the model, corresponding to transient and turbulent states of the boundary layer.

In the present work, heat-transfer experiments were carried out in a supersonic ($M = 5.0$) aerodynamic tube with an axisymmetric nozzle of diameter 0.29 m. The model (Fig. 1) was in the form of a tapering hollow cylinder (length 1.15 m; diameter 0.04 m; wall thickness 0.002 m) of 1Kh18N9T stainless steel. To allow heat-flux measurements at two cross sections of the model (I, $x = 0.3$ m from the nozzle; II, $x = 1$ m from the nozzle), Chromel—Alumel thermocouples of thickness 0.0002 m were welded to the inside of the model wall. The model was attached to a fixed mount in the tube. The tests were carried out for unsteady conditions of tube operation, associated with tube startup and with transient processes due to temperature variations of the incoming flow (between 300 and 500°K). The pressure in the tube antechamber was held constant ($P_{0a} = 8 \cdot 10^5$ N/m^2) by means of an automatic choke. The gas-flow stagnation temperature was recorded using a thermocouple assembly in the tube antechamber. The thermal inertia of the thermocouple assembly was determined experimentally and taken into account in the analysis of the test results by an appropriate correction in the test-result processing program.

The test results were processed by numerical methods using an algorithm for the solution of the one-dimensional linear inverse heat-conduction problem. In this case, the unsteady heat flux $q(\tau)$ at the surface of the model is determined by an integral Volterra equation of the first kind:

$$f_\delta(\tau) = \int_0^\tau q(\xi) K(\tau, \xi)\, d\xi,$$

where $f_\delta(\tau)$ is a known function of the initial delta. For a plane plate with a heat-insulated inner wall and constant initial temperature,

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